Regulation R20 *Subject Code:3B3DA*

TKR COLLEGE OF ENGINEERING AND TECHNOLOGY

(Autonomous, Accredited by NAAC with 'A' Grade)

B.Tech III Semester Supplementary Examinations, July 2022

COMPLEX VARIABLES AND VECTOR CALCULUS

ECE

Maximum Marks: 70 Date:19.07.2022 Duration: 3 hours

Note: **1.This question paper contains two parts A and B.**
2. Part A is compulsory which carries 20 marks. Answer all questions in Part A.
3. Part B consists of 5 Units. Answer any one full question from each unit.
4. Each question carries 10 marks and may have a, b, c, d as sub questions.

Part-A

All the following questions carry equal marks (10x2M=20 Marks)

1 Show that $u(x,y) = x^3 - 3xy^2$ is harmonic

2 Using Milne-Thomson method find $f(z)$ given that $f'(z) = 3x^2 - 3y^2 + 6ixy$

3 Evaluate $\int_c z^2 dz$ where c is the straight line segment from O(Z=0) to A(Z=2+i).

4 Expand e^z about z=0in Taylor's series expansion

5 Classify the nature of the function f(z) = $\frac{z - \sin z}{z^5}$

6 Find the residue of $e^z z^{-5}$ at z=0

7 Define gradient of a scalar point function.

8 If $\overline{f}=(x^2y)\,\bar{\imath}+(2y^2z)\bar{\jmath}+(3z^2y)\bar{K}$ then find curl $\overline{f}$.

9 State stoke's theorem.

10 Define surface integral.

Part-B

Answer All the following questions. (10M X 5=50Marks)

11 If $f(z) = \frac{1}{2}\log\left(x^2+y^2\right) + i\tan^{-1}\left(\frac{px}{y}\right)$ is an analytic function, then find the value of p. [10]

OR

12 If f(z) is a Regular function of z, then prove that $\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right]|f(z)|^2 = 4|f'(z)|^2$ [10]

13 State and prove Cauchy's integral formula. [10]

OR

14 Evaluate the integrals around c:|z-1|=3 i) $\int_c \frac{e^z}{(z+1)^2} dz$ ii) $\int_c \frac{e^z}{(z+1)^4} dz$ [10]

15 Evaluate the integrals $\int_c \frac{z-3}{z^2+2z+5} dz$ where 'c' is the circle given by [10]
i) $|Z| = 1$ $ii) |z + 1 - i| = 2$

OR

16 Find the poles and the corresponding residues of $\frac{1}{(z^2-1)^3}$ [10]

17 Prove that $div(\emptyset\bar{a}) = (grad\emptyset).\bar{a} + \emptyset\, div\bar{a}.$ [10]

OR

18 Show that vector $(x^2 - yz)\,\bar{\imath} + (y^2 - zx)\bar{\jmath} + (z^2 - xy)\bar{K}$ is irrotational and find its scalar potential. [10]

19 Verify stokes theorem for F = y^2 I + y j – 3 x k and S is the upper half ofthe sphere $x^2 + y^2 + z^2 = a^2$ and z ≥0 . [10]

OR

20 Find the work done by $\bar{f} = 3x^2\bar{\imath} + \bar{\jmath} + 2z\,\bar{k}$ along the straight line from (0,0,0) to (2,1,3). [10]